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NONLINEAR FORCED VIBRATIONS IN A HELMHOLTZ RESONATOR

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The linear theory of a Helmholtz resonator — a vessel with a short open neck — was developed by Helmholtz and Rayleigh. In this theory [1], a Helmholtz resonator is treated as a vibrating system with one degree of freedom (for the fundamental longitudinal mode), and in the first approximation it is assumed that all its kinetic energy is concentrated in the moving gas in the neck and in a certain neighborhood of the neck opening, and the potential energy of elastic deformation is in the gas in the vessel. The Helmholtz resonator is distinguished by its high Q, which is responsible for its wide use in acoustics. The process characterized by a periodic directed ejection of gas from a Helmholtz resonator with a subsequent suction of a new portion of gas from the space surrounding the neck inlet determined its technical use in devices providing pulsed periodic combustion of fuel [2, 3]. The Helmholtz resonator can clearly be used in other devices also, in which pulsed periodic physical and chemical reactions and technological processes occur with the release of energy in gas mixtures, for example in a pulsed periodic gas laser. For a technical device it is important to intensify the process; in a Helmholtz resonator this involves the excitation of strong intrinsically nonlinear vibrations [2, 3] in which the flow velocity in the neck turns out to be comparable with the velocity of sound. For such devices it is necessary to know the variation in the flow rate and the intensity of mass transfer in the neck, which also determines their efficiency. It is clear that such information can be obtained for nonlinear vibrations only by numerical methods.

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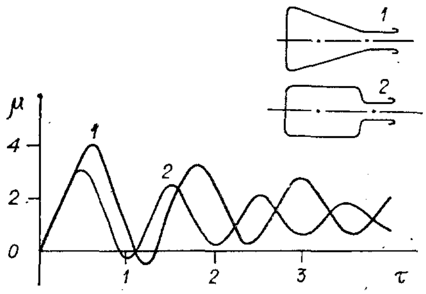

Fig. 1

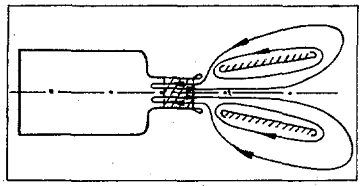

Fig. 2

We describe unsteady gasdynamic processes in a Helmholtz resonator in the quasi-one-dimensional approximation

$$\partial\bar{u}/\partial t + \partial\bar{F}/\partial x + \bar{H} d \ln A/dx = 0, \tag{1}$$

where

$$\bar{u} = [\rho,\ \rho u,\ E]^{\mathrm{T}};\ \bar{F} = [\rho u,\ p + \rho u^2,\ \rho u(p/\rho + E)]^{\mathrm{T}};\ \bar{H} = [\rho u,\ \rho u^2,\ \rho u(p/\rho + E)]^{\mathrm{T}};\quad E = \varepsilon + u^2/2;$$

Here A is the cross-sectional area of the channel and the remaining notation is standard. This system of equations is supplemented by the equation of state $p = \rho RT$ and $\varepsilon = c_V T$.

The numerical solution of Eqs. (1) with appropriate initial and boundary conditions presents no particular difficulty. We used the method described briefly in [4]. In an ideal Helmholtz resonator commonly used in acoustics $|d \ln A/dx| \gg 1$ at the junction of the neck with the vessel. This significantly complicates the computational algorithm and necessitates joining the solution in this region. We simplified the algorithm by assuming $|d \ln A/dx| < 1$. This enabled us to use the straight-line algorithm in [4]. It should be noted that, in technical devices, smooth contours of the flow region are used to reduce losses. This is evidently valid also for a Helmholtz resonator used in technical devices. Consequently, our assumption is amply justified.

As an example, Fig. 1 shows the results of a numerical study of the dynamics of the discharge of a gas through the neck for Helmholtz resonators with differently shaped vessels. The vessels have the same volume and neck geometry. The calculation was performed for an initial pressure $p_1 = 1.4 \cdot 10^4$ N/m² in the resonator cavity, and an ambient pressure $p_0 = 1.2 \cdot 10^4$ N/m². In Fig. 1 the ordinate is $\mu = m/m_n$, where $m = \int_0^t \rho u_n S_n dt$ is the flow rate of the gas through the neck, and $m_n = \rho_0 L_n S_n$ is the mass of gas filling the volume of the neck at $p = p_0$. The abscissa is the dimensionless time $\tau = ft$, where f is the natural frequency of the Helmholtz resonator given by the Rayleigh formula $f = (a_0/2\pi)\sqrt{S_n/L_n\Omega}$, where Ω is the volume of the vessel.

Figure 1 shows that during the first half-period resonator No. 1 ejects at most about 1.5 times as much gas as resonator No. 2, whereas the frequency of processes in resonator No. 1 is about 1.3 times lower. This is in qualitative agreement with results in [5], where the theory of Helmholtz resonators is developed, and is due to the larger inertia of the resonator with smoothly varying geometry at the junction of the neck with the vessel. It should be noted that there is more than one exchange of gas in the neck of resonators even during a quarter half-wave, and the velocity of the gas reaches about 200 m/sec, i.e., the vibrations in the resonator are intrinsically nonlinear. Nevertheless, the natural frequency of both resonators is in good agreement with the linear theory [1, 5].

Figure 2 shows a schematic diagram of a device in which pulsed periodic operation is maintained by the excitation of forced vibrations in a Helmholtz resonator. Here a pulsed source of thermal energy is located in the resonator neck. This device operates in the following way:

first strong vibrations are excited in the resonator in some way, for example as shown in Fig. 1;

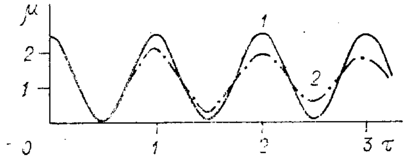

Fig. 3

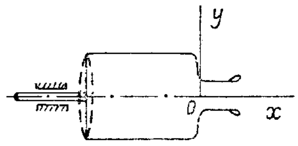

Fig. 4

if now there is a pulsed periodic supply of thermal energy with a frequency near the natural frequency of the resonator, undamped vibrations can be established with an intense exchange of gas in the neck;

the gas mixture ejected from the resonator enters the gas collector and is sucked through the gap between the resonator neck and the gas collector, resulting in a periodic restoration of the gas mixture to the region where the thermal energy is released.

It is clear that stable operation of a device with a Helmholtz resonator requires the efficient exchange of the gas in the working region of the neck. The efficiency of mass transfer, characterized by the quantity $\mu_0 = m_n/m_0$, where $m_n = 0.5 \int_{t_0}^{t_0+T} \rho|u|S_n dt$ is the mass of gas sucked in or ejected in a period $T = f^{-1}$, and $m_0 = \rho_0 S_n L_0$ is the mass of gas filling the working region of the neck, depends on the resonator geometry, the dimensions of the working region, and the specific energy input. As an example, Fig. 3 shows the dependence of μ on t for optimal (curve 1) and nonoptimal (curve 2) resonator geometry. It is clear from this that in the second case undamped ($\mu_0 > 1$) vibrations are not realized for the given energy input.

Alternative calculations of nonlinear vibrations in a Helmholtz resonator are rather laborious. Therefore, it is expedient to use numerical methods to select the design parameters. In what follows we derive rather simple relations for determining the basic parameters of a Helmholtz resonator in the case of nonlinear vibrations. These relations were obtained by using known facts and the results of our numerical studies: 1) the wavelength of natural vibrations in a Helmholtz resonator is much longer than its linear dimensions and, therefore, we assume that the flow velocity in the neck is constant along its length, i.e., $\partial u/\partial x \approx 0$; 2) the flow velocity in the vessel is negligibly small, i.e., the pressure in the vessel is uniform over its volume; 3) the flow velocity in the neck varies almost sinusoidally, i.e., we can describe the time dependence of the velocity by $u = u_0 \sin \omega t$; the nonlinearity has only a small effect on the natural frequency.

Then the equation of motion of the gas in the neck can be written in the form

$$\rho \partial u/\partial t + \partial p/\partial x + \xi \rho u|u|/2D_e = 0. \tag{2}$$

Here the last term takes account of the frictional resistance on the neck walls. We write the boundary condition at the edge of the neck in the form

$$p(t) = p_0 + (\rho u/4)(|u| - u),$$

where the last term takes account of the dynamic head in the influx of gas into the neck.

Taking $\rho = \rho_0$ as the first approximation, we obtain the solution of Eq. (2) in the following form (for the junction of the neck with the vessel):

$$p(t) = p_0 + 0.5\rho_0 u_0^2 [2u_0 \omega L_n \cos \omega t + ((\xi L_n/D_e)|\sin \omega t| + 0.5(|\sin \omega t| - \sin \omega t)) \sin \omega t].$$

From this we obtain an expression for the work done in forcing gas through the neck during one period:

$$A_p = S_n \int_{t_0}^{t_0+T} p u dt = \frac{4}{3} \frac{\rho_0 u_0^3}{\omega} (0.5 + \xi L_n/D_e) S_n.$$

Here the first term in parentheses corresponds to the kinetic energy losses in the ejection of gas from the neck, and the second term describes the frictional losses on the neck walls. It is clear that for neck dimensions typical for a Helmholtz resonator ($L_n/D_e < 10$) the principal losses result from the ejection of gas from the neck. This conclusion was used in the numerical studies.

Suppose the vibrations of the gas in a Helmholtz resonator are maintained by pulsed periodic heat release in the working region of the neck L_0. The total work performed by the heated gas in adiabatic expansion is

$$A_e = \rho_0 c_V S_n L_0 (T_W - T_0'),$$

where T_W is the temperature of the gas immediately after the energy release and T'_0 is the temperature of the gas after adiabatic expansion to the initial pressure.

Assuming that only one k-th of the work done in the expansion of the heated gas compensates the expenditure of energy in forcing the gas through the neck, we obtain

$$\mu_0 \simeq \left[\frac{6kL_n\,\Omega}{S_n L_0^2 \gamma(\gamma-1)(0.5+\xi L_n/D_e)}\right]^{1/3} [1+(1+\gamma)W-(1+(1+\gamma)W)^{1/\gamma}],$$

where $W = E_T/(\rho_0 S_n L_0 c_V T_0)$ is the relative energy input.

Hence, by assuming $\mu_0 > 1$, we can determine either the Helmholtz geometry or the required value of W.

In a number of cases the value of W is limited (e.g., in a CO_2 laser), and may be insufficient to excite vibrations and to ensure the necessary mass transfer. Therefore, it is of practical interest to consider a scheme with external excitation of nonlinear vibrations. One such scheme in which the volume of the vessel is varied is shown in Fig. 4.

We describe the operation of such a device with a mobile rear wall (Fig. 4) in the acoustic approximation [1]. Then the variation of the flow parameters in the neck is described by the following system of equations:

$$\frac{\partial u}{\partial t} + \frac{1}{\rho_0}\frac{\partial p}{\partial x} + \alpha_0 u = 0, \quad \frac{\partial p}{\partial t} + \rho_0 \frac{\partial u}{\partial x} = 0. \tag{3}$$

where α_0 takes account of frictional and radiation losses.

Assuming that the pressure is uniform over the volume of the resonator vessel and that the mobile wall is vibrated harmonically, i.e., the volume of the vessel varies according to the law $\Omega = \Omega_0(1 + \nu \exp i\omega t)$, we obtain the solution of Eqs. (3) in the form:

for the velocity of the gas in the neck,

$$u(x, t) = \nu \frac{\exp \lambda(\bar{x}-1) + \exp \lambda(1-\bar{x})}{(\lambda-\varepsilon)\exp(-\lambda) - (\lambda+\varepsilon)\exp\lambda} \exp i\omega\tau; \tag{4}$$

for the pressure in the vessel,

$$p(t) = p_0 + \chi \frac{\exp\lambda - \exp(-\lambda)}{(\lambda-\varepsilon)\exp(-\lambda) - (\lambda+\varepsilon)\exp\lambda} \lambda \exp i\omega\tau, \tag{5}$$

where $\lambda = \sqrt{ik\bar{\omega} - \bar{\omega}^2}$, $\chi = \nu\rho_0 a^2_0$ is the amplitude of the pressure variations in a closed vessel, $k = \alpha_0 L_n/a_0$, and $\varepsilon = S_n L_n/\Omega_0$. In writing solutions (4) and (5) we have used the following dimensionless quantities:

$$\bar{x} = x/L_n, \quad \tau = ta_0/L_n, \quad \bar{\omega} = \omega L_n/a_0.$$

We can obtain from this the amplitudes of the pressure variations in the vessel, the phase shift of the vibrations, and the resonance frequencies. For $k \ll \bar{\omega}$, we have, for the first harmonic, $\omega_0 \approx a_0\sqrt{S_n/L_n\Omega_0}$, which agrees with [1, 5], and the amplitude of the pressure variations is given by

$$\Delta p \simeq \rho_0 a_0^2 \nu (1 + 0.33\varepsilon)(1 + \varepsilon/k^2).$$

Since the total venting of the volume of the neck requires a pressure change $\Delta p \approx \rho_0 a^2_0 \varepsilon$ in the vessel, we obtain for ν,

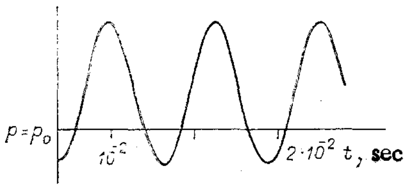

Fig. 5

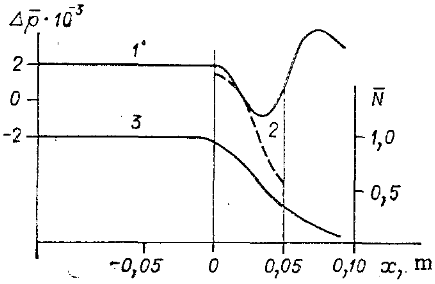

Fig. 6

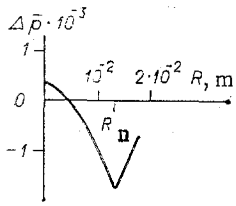

Fig. 7

$$\nu = 0.5\alpha_0 L_n \sqrt{\varepsilon}/a_0.$$

Numerical studies showed that the natural frequency of nonlinear vibrations of a Helmholtz resonator practically coincides with $\omega_0 = a_0\sqrt{S_n/L_n\Omega_0}$, and the amplitude of the pressures in the vessel is in good agreement with the estimates obtained by assuming that the loss factor α_0 is determined only by losses of kinetic energy of the gas jet emerging from the neck.

To study the characteristics of a Helmholtz resonator with a mobile wall, we constructed the apparatus shown schematically in Fig. 4 together with the coordinate system used in the measurements. The resonator vessel was $a \sim 10^{-3}$ m³ cylindrical flask. The neck was about $5 \cdot 10^{-2}$ m long and about $2.4 \cdot 10^{-2}$ m in diameter. Measurements showed that the natural frequency of the resonator was about 170 Hz, which is practically the same as that given by the linear theory [1, 5].

The average pressure in the resonator cavity was measured with an inertial U-tube manometer, one arm of which was connected to a long tube-probe. The dynamic component of the oscillating pressure was measured with a dynamic probe [6] whose sensitive element was a microphone. The signals were recorded by an oscillograph. Air at atmospheric pressure was used as a working substance. The results presented below were obtained at the resonance frequency of about 170 Hz.

It is clear from Fig. 5, which shows the pressure variations in the resonator vessel, that the average pressure is higher than the ambient pressure. This excess increases with increasing amplitude of the vibrations of the mobile wall. The distribution of the average relative pressure along the axis of the resonator $\Delta\bar{p}_{y=0} = (\bar{p} - p_0)/p_0$ (curve 1) and on the neck wall $\Delta p_W = (\bar{p}_W - p_0)/p_0$ (curve 2) are shown in Fig. 6 together with the distribution of the relative acoustic power (curve 3) of the pressure fluctuations

$$\overline{N}(x) = \left(\int_0^T |\Delta p(x, t)|\, dt\right)\left(\int_0^T |\Delta p(0, t)|\, dt\right)^{-1},$$

where $\Delta p(0, t)$ is the pressure in the resonator vessel, and T is the period of the vibrations.

Figure 6 shows that the average pressure $\bar{p}$ in the resonator vessel and in the surrounding space near the neck is higher than p_0, but, in the neck, particularly near its wall, the average pressure is considerably reduced, and over a large part of its length it is below atmospheric. The distribution of the average pressure along the radius at the edge of the neck is shown in Fig. 7. It should be noted that the maximum flow velocity in the neck is more than 100 m/sec.

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BORN APPROXIMATION OF THE SOLUTION OF THE INTERNAL WAVE SCATTERING PROBLEM

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Difficulties in the execution of detailed and extensive measurements of internal wave parameters in the ocean retard, to a certain extent, the development of a correctly deduced theory. In particular, little is known about the internal wave energy distribution between different modes. For a number of reasons it is considered that, for a sufficiently definite thermocline, the lowest mode will dominate, whose behavior is investigated in most detail in theoretical respects [1]. However, higher modes characterized by higher values of the transverse velocity gradient, and increase in the possibility of local instability and degeneration into turbulence, play an important part in the development of internal wave spectra. Consequently, it is of interest to examine methods of energy transmission in the internal wave spectrum. The modal structure is evidently shaped as a function of the variability of a whole series of parameters specifying the propagation law and the interaction of internal waves in the ocean. Consequently, for instance, problems of internal wave propagation in the presence of horizontal density field inhomogeneities [2, 3], shear flows [4, 5], and arbitrary vertical density field [6], etc., were examined. A sufficiently complete list of literature can be found in [7-9].

One of the possible mechanisms of internal wave energy redistribution between different modes of the scattering of internal waves by localized density field inhomogeneities is discussed. The simplest problem is formulated here: The Boussinesq approximation is used to describe a stratified fluid, and rotation of the earth is neglected while the density field inhomogeneities are considered not to vary in time and to be at rest.

Within the framework of assumptions made in the linear formulation of the problem, and neglecting molecular viscosity forces, the initial system of equations describing the dynamic state of the medium has the form [1]

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{1}{\rho_0}\nabla p + \frac{\rho}{\rho_0} g\mathbf{k} = 0, \quad \nabla\mathbf{U} = 0, \quad \frac{\partial \rho}{\partial t} + \mathbf{U}\nabla\rho_1 - \rho_0 g^{-1} N_0^2 w = 0, \tag{1}$$

where $\mathbf{U} \equiv \{u, v, w\}$ is the velocity vector of particles of the medium; p, pressure; ρ, deviation of the density from the initial density distribution, equal to $\bar{\rho}(z) + \rho_1(\mathbf{r})$, where $\bar{\rho}(z)$ is the density distribution in the absence of inhomogeneities and $\rho_1(\mathbf{r})$ is a function characterizing the density-field inhomogeneity; $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, and $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$, unit vectors along the Cartesian x, y, z coordinate axes; g, acceleration of gravity; $N_0^2 \equiv -(g/\rho_0)(d\bar{\rho}/dz)$, Vaisala-Brunt frequency. It should be noted that stationary flows generally exist for such an assignment of the density field. But since these flows are sufficiently slow, they can be neglected in a first approximation and a density field given by a function independent of the time can be considered (see [3], for example).

We will be interested below in a function $\mathbf{U}(\mathbf{r}, t)$. Consequently, we go from system (1) over to a system of equations for u, v, w that does not contain the functions p(x, y, z) and ρ(x, y, z):

$$\frac{\partial^2}{\partial t^2}\Delta w + N_0^2(z)\Delta_h w = \frac{g}{\rho_0}\Delta_h(\mathbf{U}\nabla\rho_1). \tag{2}$$

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